From String to Membrane

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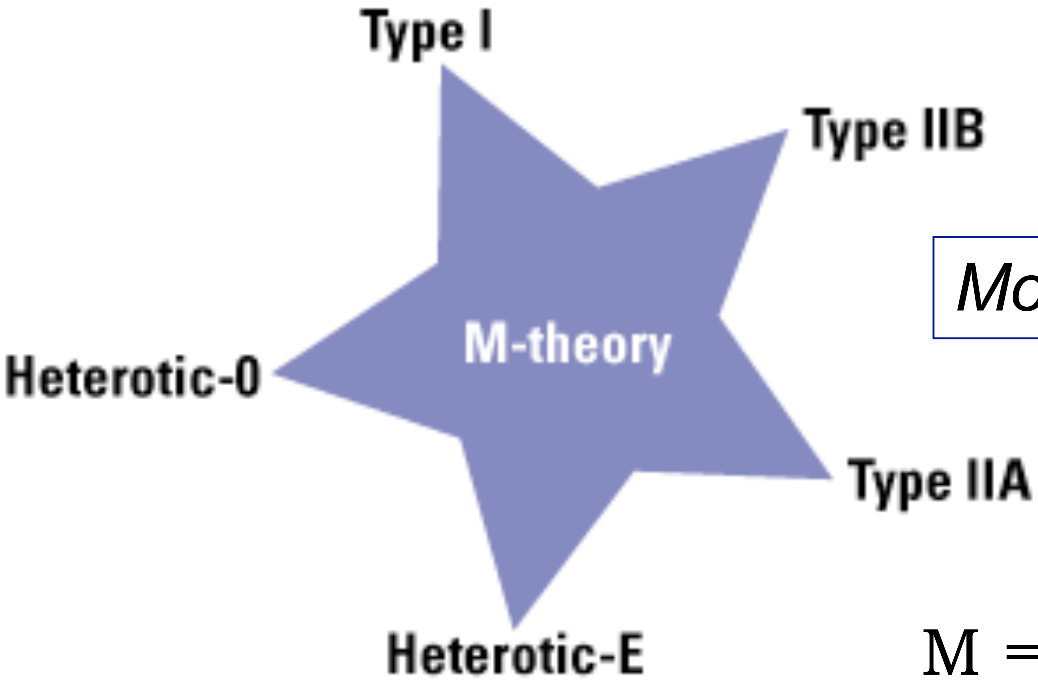
Why Membrane?

There are 5 superstring theories:

IIA, IIB, I, heterotic $E8 \times E8$, heterotic $SO(32)$



Moduli space of M theory

M = "mysterious", "mother"

Degrees of Freedom

- **String theory:**

  F1 (fundamental string) -- B field

  Dp-branes (solitons of p+1 dim)

  -- A_{p+1} (differential form of order p+1)

- **M theory**:

  M2 (fundamental membrane) -- C field

  M5 (solitons of 5+1 dim, analogous to a magnetic monopole) -- dual of C field

  There is no free parameter except M_{pl}.

Evidence

Supergravity theories:

Unique 11D SUGRA (low energy limit of M)

dimension reduction → 10D type IIA SUGRA

T-duality → 10D type IIB SUGRA on S^1

etc…

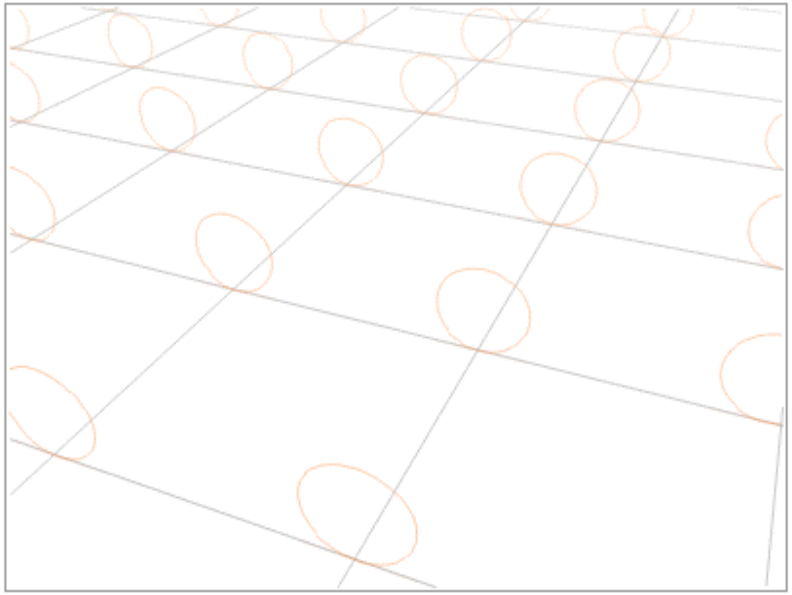
String = charge of B-field: $B_{\mu\nu} = - B_{\nu\mu}$

Dp-brane = charge of A_{p+1} field: $A_{\mu\nu\ldots\lambda}$

(a totally skew symmetric tensor field with p+1 indices)

Membrane = charge of C-field: $C_{\mu\nu\lambda}$

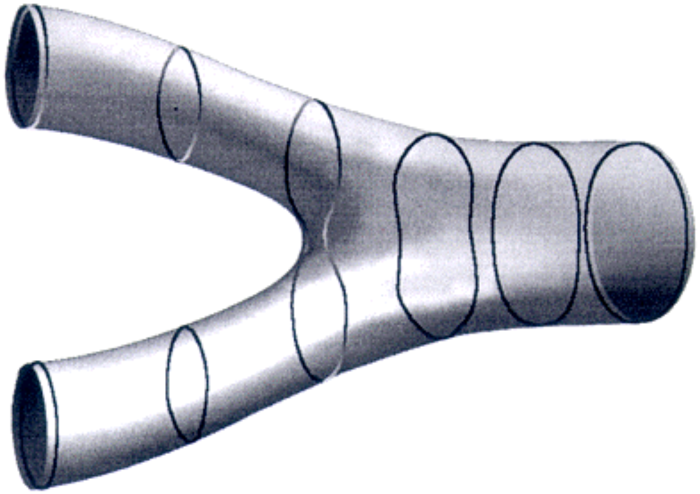
IIA/M Duality



- F1 --- M2; D2 --- M2
- D4 --- M5; NS5 --- M5
- D6, D8 --- geometric configurations

Single String (F1)

- Nambu-Goto action = area of worldsheet x the string tension
- Add fermions for supersymmetry
- We can define perturbative string theory and compute scattering amplitudes.

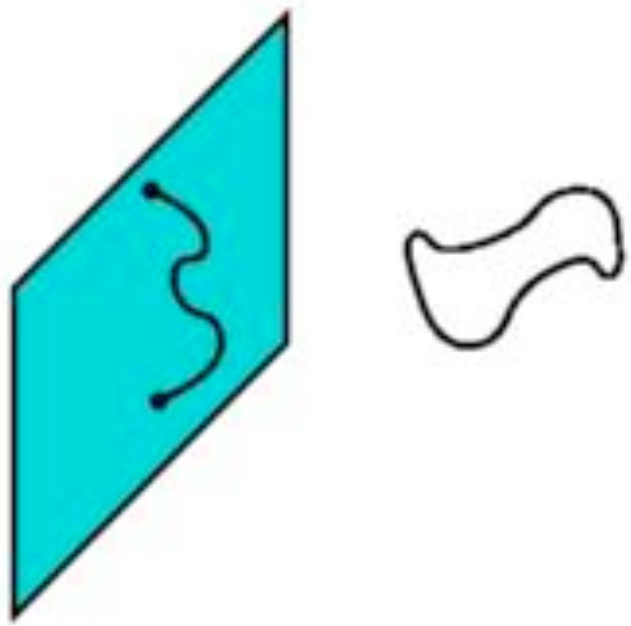


Single Dp-brane

- D.O.F. of Dp = open string on Dp
- In the low energy limit:

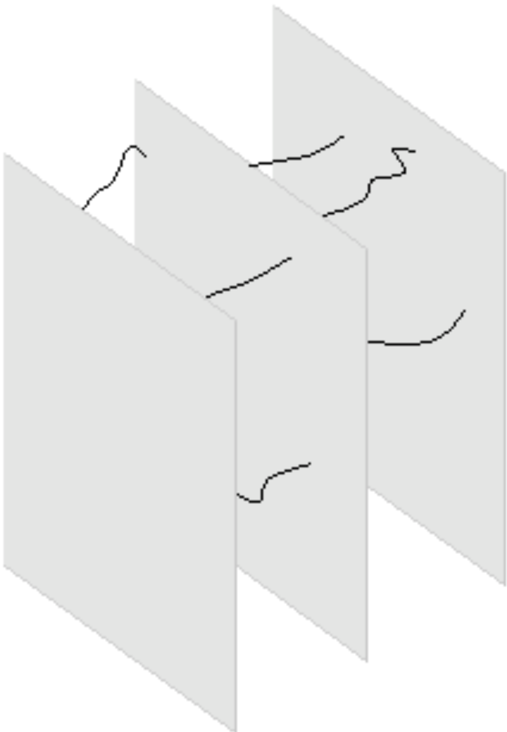
  Transverse osc. --- osc. of Dp-brane

  Longitudinal osc. --- vector field on Dp



Multiple D-branes

- Super Yang-Mills theory A_{ij}
- U(N) gauge theory with supersymmetry
- Entropy $\propto N^2$



The theories of multiple D-branes may describe string theory in certain limits:

BFSS matrix model, DVV matrix string, c=1 matrix model, etc.

Single M2, Single M5

- Generalization of Nambu-Goto action = worldvolume times tension
- Add fermions for supersymmetry.
- There is no well-defined calculation of scattering amplitudes.
- No perturbative M theory defined in the same way as the perturbative string.
- Single M2, M5-brane actions are known.

Multiple M2, Multiple M5

- What are the gauge symmetries?
- Entropy $\propto N^{3/2}$ for M2; N^3 for M5
- This has been a puzzle for a long time.
- Understanding this may help understand M theory.

Breakthrough on Multi-M2

- *Gustavsson (07):*

  A new algebraic system for gauge symmetry using Lie 3-algebra.

- *Bagger, Lambert (06, 07):*

  An $\mathcal{N} = 8$ superconformal field theory in 2+1 dimensions

  Lie 3-algebra for gauge symmetry

  No free parameters

Lie 3-algebra and membrane

- string » matrices » Lie algebra
- membrane » ??? » Lie 3-algebra

  [Bagger-Lambert-Gustavsson (BLG) model]

Multiple Membrane Lagrangian [Bagger-Lambert]

$$L = \frac{1}{2}\langle (D_\mu X^I)^2 \rangle - \frac{1}{12}\langle [X^I, X^J, X^K]^2 \rangle + \text{fermions}$$

$$\mu = 0, 1, 2; I, J, K = 3, \cdots, 10.$$

Lie 3-algebra

- $H = \{ T^a \}$, $[A, B, C] \in H$
- $[A, B, C]$ = tri-linear, skew-symmetric
- $[A, B, [C, D, E]] = [[A, B, C], D, E] +$

  $[C, [A, B, D], E] + [C, D, [A, B, E]]$

  fundamental id. (generalized Jacobi id.)
- Let $\delta O = [A, B, O]$

  $\delta[C, D, E] = [\delta C, D, E] + [C, \delta D, E] + [C, D, \delta E]$

$\mathcal{A}_4$

- 4 generators $\{T_a\}$ $(a = 1,2,3,4)$
- $[T_a, T_b, T_c] = \varepsilon_{abcd} T_d$

  [Filippov 85: n-Lie algebras]

- generalization of *su(2)*
- Symmetry transformation is generated by two generators:

  $\delta A = \Lambda_{ab} [T_a, T_b, A]$

  = infinitesimal SO(4) rotation

Nambu-Poisson bracket

- Generalization of Poisson brackets [Takhtajan 94]

Recall: Poisson brackets are infinite dim. Lie algs.

- $\{f, g, h\} = P^{abc}\, \partial_a f\, \partial_b g\, \partial_c h$

1. Skew-symmetry
2. Fundamental identity
3. Leibniz rule:

   $\{f, g, h_1.h_2\} = \{f, g, h_1\}h_2 + \{f, g, h_2\}h_1$

- Decomposability theorem:

  Locally $P^{abc} = \varepsilon^{abc}$ for 3 of the n coordinates

Lie n-algebra [Filippov 85]

- $H = \{ T^a \}$, $[A, B, \ldots, Z] \in H$
- $[A, \ldots, B]$ = multi-linear, skew-symmetric
- $[A, \ldots, B, [C, \ldots, D]] = [[A, \ldots, B, C], \ldots, D] + \ldots + [C, \ldots, [A, \ldots, B, D]]$ (fund. id.)
- Transformation: $\delta C = [A, \ldots, B, C]$
- The fundamental id. ensures that this transformation generates a Lie algebra.

Symmetries of the BLG model

- There is (almost) no free parameter.
- SO(2,1) x SO(8)
- $\mathcal{N} = 8$ SUSY
- Gauge symmetry generated by Lie 3-alg:

  $\delta X = \Lambda_{ab} [T_a, T_b, X]$

  gauge potential $A_{\mu ab}$

Problems

- Very little is known about Lie 3-algebra.
- Very few examples [only $\mathcal{A}_4$ until Ho-Hou-Matsuo 08].
- Metric (Killing form) is not positive-definite except $\mathcal{A}_4$ and trivial algebra. [Ho-Hou-Matsuo 08, Papadoupoulos 08, Gauntlett 08, Gutowski 08]
- negative-norm $\Rightarrow$ ghost
- $X = X_a\, T_a$

$$\langle (\partial_\mu X)^2 \rangle = (\partial_\mu X_a) \langle T_a , T_b \rangle (\partial_\mu X_b)$$

M2 to D2

- M2 $\rightarrow$ D2 [Gomis-Milanesi-Russo, Benvenuti-Rodriguez-Gomez-Tonni-Verlinde, Ho-Imamura-Matsuo, 2008]
- Exact D2-brane action (U(*N*) SYM) was obtained [Ho-Imamura-Matsuo 08].
- Correct interpretation of dimension reduction avoids ghosts [Ho-Imamura-Matsuo 08].

M2 to M5

Nambu-Poisson as Lie 3-algebra $\Rightarrow$

M5-brane low energy theory: (X, A)

[Ho-Matsuo, Ho-Imamura-Matsuo-Shiba 08]

X = transverse coordinates (embedding)

A = 2-form gauge potential with

self-dual 3-form field strength:

$$F_{abc} = \partial_a A_{bc} + \partial_b A_{ca} + \partial_c A_{ab}$$

$$F_{abc} = \varepsilon_{abcdef} F_{def}$$

Entropy for multiple M2

Lie 3-algebra by truncation of Nambu-Poisson algebra ⇒ entropy of M2 proportional to $N^{3/2}$.
[Chu-Ho-Matsuo-Shiba 08]

SUSY algebra of M theory

Central element added to Lie 3-algebra ⇒ topological central charges of SUSY algebra.
[Furuuchi, Takimi, Shih 08]

CONCLUSION

- A lot remains to be learned about Lie 3-algebra, and Lie n-algebras:

  more examples, classifications, representations, construction of reps., Lie 3-group, topological issues, ...

- Multiple M5-brane theory?
- "Symmetry dictates interactions".

  Are there other physical systems with symmetries of Lie 3-(n-)algebras?

- Phenomenological applications?